Abstract – This paper analyzes the effects caused by the unsymmetrical voltage sags in the induction machine supply system: current and torque peaks, and mechanical speed loss. These effects depend on many elements, such as the sag magnitude and duration, the type of sag (symmetrical or unsymmetrical), and the fault and recovery voltage instants. It is shown that the most severe transient occurs for specific instants. Extensive ranges of voltage drop have been analyzed (for different magnitude and duration and the most severe fault and recovery voltage instants), and machine sensitivity is graphically shown in CBEMA curves. These curves can be applied to protective relay coordination, especially for large machines. It is also shown that different unsymmetrical sags with the same positive-sequence voltage produce similar effects. These unsymmetrical sags can be grouped to express machine sensitivity in CBEMA curves, using the positive-sequence voltage for ordinates.

Index Terms – unsymmetrical voltage sag, sensitivity curves, CBEMA curves, induction machine.

I. INTRODUCTION

In a first approximation, a voltage sag (dip) is a reduction between 10% and 90% in rms voltage, with a duration from 0.5 cycles to 1 minute [1]. A short circuit or overload on the utility system generally originates it. Typically, the voltage sag duration ranges from 0.5 to 30 cycles, and the magnitude depends on the power system distribution and the proximity to the fault site. (In this paper, the sag magnitude is the net rms voltage in percent or per unit of system nominal voltage).

The voltage at the customer bus is transient during the fault and after clearing the fault (that means voltage is non-sinusoidal). For simplicity purposes, the shape of the sag is considered rectangular. Then, it can be characterized by a duration and a depth, as well as by the possible associated phase angle shift (phase jump) [2].

Voltage sags can be either balanced or unbalanced, depending on the causes. If the individual phase voltages are equal, the sag is balanced. If the individual phase voltages are different or the phase relationship is other than 120°, the sag is unbalanced. A three-phase short circuit or a large motor starting can produce symmetrical sags. Single-line-to-ground (SLGF) or phase-to-phase faults due to lightning, animals, accidents, etc. can cause unsymmetrical sags. The large majority of faults in the utility system are SLGF's. Three-phase faults are more severe but much less common [3]. The load and the transformer connections influence the sag. A classification of the sags is included in [2].

Different categories of industrial equipment have different sensitivities to voltage sags [3]. The main categories of sensitive loads are motors, adjustable-speed drives, another type of power electronic equipment, discharge lamps and control devices –computers, programmable logic controller and contactors–.

The consequences of a voltage sag in the induction machine supply are speed loss and current and torque peaks that appear in the voltage drop and recovery points [4,5]. This transient can trigger the motor or system protections.

In this paper, the effects of unsymmetrical voltage sags on induction machines are analyzed and compared with the effects of the symmetrical sags. Section II shows a voltage sag classification. The induction machine model is included in Section III and used in Section IV to study the behavior of the machine for the different sag types. The influence of the fault and recovery instant is discussed in Section V. Section VI includes CBEMA curves to show graphically machine sensitivity to the different sag types. These curves plot current and torque peaks, and speed loss versus voltage dip. The use of positive-sequence voltage to study the effects of unsymmetrical sags on the machine is also suggested. An application of the CBEMA curves is shown.

II. VOLTAGE SAG TYPES

In [2] it is shown that voltage sags experienced by three-phase loads can be classified into four types, denoted as A, B, C and D. Fig. 1 shows their phasor diagrams: dotted lines are the pre-fault voltage phasors and solid lines are the fault voltage phasors.

Sag type A is balanced. All phasors drop the same amount in magnitude. They can be defined by (1), where \( h \) is the sag magnitude or depth (0 \( \leq h \leq 1 \)).

\[ V_{\text{p}} = V_{\text{f}} (1 - h) \]

\[ V_{\text{f}} = V_{\text{p}} \]
\[ V_a = hV \]
\[ V_b = -\frac{1}{2}hV - j\frac{\sqrt{3}}{2}hV \]  
(1)
\[ V_c = -\frac{1}{2}hV + j\frac{\sqrt{3}}{2}hV \]

Types B, C and D are unsymmetrical sags. For sag type B only one phasor drops in magnitude. For sag types C and D, two phasors drop in magnitude and change in phase angle. For sag type D the third phasor only drops in magnitude. Sag types B, C and D are defined by (2), (3) and (4):

\[ V_a = V \]
\[ V_b = -\frac{1}{2}V - j\frac{\sqrt{3}}{2}V \]  
(2)
\[ V_c = -\frac{1}{2}V + j\frac{\sqrt{3}}{2}V \]
\[ V_a = V \]
\[ V_b = -\frac{1}{2}V - j\frac{\sqrt{3}}{2}hV \]
\[ V_c = -\frac{1}{2}V + j\frac{\sqrt{3}}{2}hV \]  
(3)
\[ V_a = V \]
\[ V_b = -\frac{1}{2}hV - j\frac{\sqrt{3}}{2}V \]
\[ V_c = -\frac{1}{2}hV + j\frac{\sqrt{3}}{2}V \]  
(4)

III. INDUCTION MACHINE MODEL

The chosen model for the induction machine is a linear single-cage transient model. When the Ku transformation [6] in the synchronously reference frame is applied, the transformed equations are:

\[ s = \frac{\omega_q - \omega_m}{\omega_q} \]
\[ \Gamma \approx 2M \text{Im} e^{j\phi} \]

IV. VOLTAGE SAG EFFECTS

The transient shape depends on many elements, such as the sag magnitude and duration, the electrical parameters of the machine, the load and the mechanical inertia. Depending on the type of fault (balanced or unbalanced), the fault and recovery voltage instants (or the corresponding voltage angles) have different effects on current and torque peaks.

Irrespective of the type of sag, the observed effects of a voltage sag on the induction machine supply are speed loss and current and torque peaks.

Fig. 2, Fig. 3, Fig. 4 and Fig. 5 show the machine behavior for sag types A, B, C and D, respectively. The sags have a depth \( h = 0.1 \) with 300 ms of duration. The chosen motor for the simulation is an actual motor that drives a ventilator in a cement plant: 610 kW, 3300 V (star), 50 Hz, 7850.5 Nm, 742 \( \text{min}^{-1} \) and 148 A. Its operating point is close to the nominal conditions.
The effects of sag type A, Fig. 2, have been studied in [4,5]. Mechanical speed loss shows initial oscillations in the voltage drop and recovery points, points 1 and 2. Current and torque peaks are usually obtained in the first cycle after the drop or the recovery voltage points, but in certain conditions in the next 2 or 3 cycles. The fault and recovery voltage instants have minimal influence in current peak and no influence in torque peak and speed loss.

The transients of unsymmetrical sags are very different from the symmetrical ones. They are sensitive to the fault and recovery voltage instants, or voltage angles in these instants. This influence is studied in Section V. On the other hand, transient speed and torque show non-damped oscillations because of the negative-sequence voltage. By comparing symmetrical and unsymmetrical sags with the same depth – in (1), (2), (3) and (4) – and duration, it can be observed that current peaks are usually lower in the unsymmetrical sags whereas torque peaks can be higher.

Among the unsymmetrical sags, type B produces lower current and torque peaks, and speed loss, Fig. 3.

Sag types C and D, Fig. 4 and Fig. 5, with the same depth and duration, show similar transients. Similar torque and current peaks can be expected. This will be verified in the CBEMA curves of Section VI.

V. INFLUENCE OF THE FAULT AND RECOVERY VOLTAGE INSTANTS

If the fault is balanced, type A, the fault and the recovery voltage instants (or the corresponding voltage angles) have less influence in current peak and no influence in torque peak and speed loss. They only cause current peak to be obtained in one or another phase [4,5]. However, these instants have a great influence in current and torque peaks when the fault is unbalanced, and no influence in speed loss.
It can be observed that speed loss is independent of phase voltage. The corresponding plots for sag of types C and D are similar.

Similar results have been obtained for sags with different depth and duration, and for machines of a wide range of power.

Therefore, when unsymmetrical voltage sags are applied to an induction machine, the maximum current and torque peaks are obtained for specific phase voltages: 90° for sag of types B and D, and 0° for sag of type C.

**VI. SENSITIVITY CURVES**

An algorithm has been developed to analyze extensive ranges of voltage drop (for different magnitude and duration and the most severe fault instant).

Machine sensitivity is graphically shown in the following CBEMA curves for speed loss, Fig. 9, current peak, Fig. 10, and torque peak, Fig. 11. The axes of these curves represent...
resent the magnitude, \( h \), and the duration—in logarithmic scale—of the events.

These curves display the results of 4800 voltage sags: 120 different sag durations (from 1 ms to 10 s) and 40 magnitudes (from 0% to 97.5%). The transients have been simulated for the unfavorable voltage angles mentioned in Section V. Displayed current and torque peaks are the maximum values between the peak of the voltage drop point and the peak of the voltage recovery point.

### A. Curves shape

With reference to mechanical speed loss, the most severe sag is type A, Fig. 9a. Types C and D, Fig. 9c and Fig. 9d, have similar effects, and type B, Fig. 9b, is the least severe type.

The maximum current peaks are usually obtained in sag type A, Fig. 10a. Sag types C and D, Fig. 10c and Fig. 10d, have similar curves. In some cases, these unsymmetrical sags can produce higher peaks than the symmetrical sag, as

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Fig. 12. CBEMA curves of instantaneous current peak for sag types B, C and D, using the positive-sequence voltage for ordinates

Fig. 13. CBEMA curves of instantaneous torque peak for sag types B, C and D, using the positive-sequence voltage for ordinates
is the case of point $P$ (Fig. 10).

Sag of types C and D, Fig. 11c and Fig. 11d, can produce higher torque peaks than type A, Fig. 11a, as is the case of point $Q$.

With reference to current and torque peaks, sag type B, Fig. 10b and Fig. 11b, is also the least severe type of sag.

When observing the curves, sag of types C and D could be represented with the same CBEMA curve.

B. Sensitivity curves using positive-sequence voltage for ordinates

As the machine behavior depends on the type of sag, it would be interesting to find a parameter that defines the effects for the four types of sag.

This parameter could be positive-sequence voltage during the sag. It is necessary therefore to study the consequences of different sag types with equal positive-sequence voltage.

The Fortescue transformation (symmetrical components transformation), allows positive-, negative-, and zero-sequence voltages to be calculated from real phase voltages:

\[
\begin{align*}
\frac{1}{3} & \quad a' = a \\
\frac{1}{\sqrt{3}} & \quad a'' = \frac{a}{\sqrt{3}} \\
\frac{1}{\sqrt{3}} & \quad a'' = \frac{a}{\sqrt{3}}
\end{align*}
\]

By using the expressions of the four sag types, (1), (2), (3) and (4), and neglecting the zero-sequence voltage (induction motors are usually connected either delta or ungrounded wye):

\[
\begin{align*}
V_{1,A} &= h V \\
V_{1,B} &= \frac{2+h}{3} V \\
V_{1,C} &= \frac{1+h}{2} V \\
V_{1,D} &= \frac{1+h}{2} V
\end{align*}
\]

Equation (7) shows that minimum positive-sequence voltage in sag type B is 2/3-V; in sag types C and D is 1/2-V. Maximum negative-sequence voltage in sag type B is 1/3-V; in sag types C and D is 1/2-V.

Fig. 12 and Fig. 13 show CBEMA curves of current and torque peaks for sag of types B, C and D, using positive-sequence voltage for ordinates. The graphics for sag type A are Fig. 10a and Fig. 11a, because sag is symmetrical ($V_a = V_{1,A}$, (1), (7)).

Mechanical speed loss has not been plotted versus positive-sequence voltage because it also depends on the negative-sequence voltage. Remember that negative-sequence currents produce negative average electrical torque, and electrical torque is related to speed loss.

Except for small differences, the curves of the three unsymmetrical sags agree. Thus, if CBEMA curves are plotted using positive-sequence voltage for ordinates, the study of voltage sags can be simplified to two typologies: symmetrical and unsymmetrical sags.

C. Application of the curves

The CBEMA curves can be applied to protective relay coordination (machine or system protection calibration). In this sense, an intermediate solution must be chosen between the process requirements and the security of either the installation or the motor.

Fig. 14a shows one isoline of the current peak CBEMA curves for each sag. These isolines correspond to four times $\sqrt{2}I_N$. By setting the protections to trip at four times $\sqrt{2}I_N$, the machine or the system are immune to symmetrical and unsymmetrical sags included in shadow areas of Fig. 14b.

D. Developed algorithm. Calculation time

The algorithm developed to analyze extensive ranges of voltage drops has a high calculation speed because it makes use of previous calculations when it is possible.
Sensitivity curves in this paper display the results of 4800 voltage sags: 120 different sag durations and 40 magnitudes. Their calculation time is about 90 minutes in a Pentium MMX 300 MHz and 64 Mb of RAM computer.

VII. CONCLUSIONS

Fault and recovery voltage instants (or phase voltages) influence current and torque peaks in the unsymmetrical sags. The most severe peaks in sags of types B and D are obtained when phase voltage is 90°; in sags of type C when phase voltage is 0°.

CBEMA curves have been displayed to represent machine sensitivity to unsymmetrical voltage sags. Unsymmetrical voltage sags –types B, C and D– usually produce softer current and torque peaks than symmetrical sags with the same depth and duration. However, sags of types C and D can produce higher peaks in some cases.

As types C and D have similar CBEMA curves, they could be represented in the same graph. CBEMA curves can be applied to setting protection for the immunization of the system or the machine to extensive ranges of voltage sags.

Different unsymmetrical sags with the same positive-sequence voltage produce similar effects. Therefore, the study of voltage sags can be simplified to two typologies (symmetrical and unsymmetrical sags) if CBEMA curves are plotted using positive-sequence voltage for ordinates.

REFERENCES


Fig. 10. CBEMA curves of instantaneous current peak for sag types A, B, C and D

Fig. 11. CBEMA curves of instantaneous torque peak for sag types A, B, C and D